The removal of particulate matter is central to the drinking water treatment process. An integrated model for describing contact, direct, and nonsweep conventional filtration is incorporated into an optimization framework to determine least-cost treatment configurations and design parameters that satisfy hydraulic and effluent concentration constraints. Various influent particle concentrations, size distributions, and size ranges under different flow rates and particle densities are explored to illustrate the importance of size distribution characteristics, flow rate, and particle density on design decisions. In general, contact filtration and conventional filtration are the predominant treatment processes, with direct filtration used sparingly for waters with higher concentrations of small particles. The results illustrate that the volume average diameter is not sufficient to characterize the treatment performance; rather, the particle size range and distribution shape are also needed to determine the appropriate treatment configuration. Increasing the design flow rate (2, 10, and 75 mgd (7.57, 37.85, and 283.88 ML/d)) and particle density (1.05, 1.20, and 2.40 g/cm³) both lead to a preference for contact filtration over conventional filtration.

Treatment plant design for particulate removal: EFFECTS OF FLOW RATE AND PARTICLE CHARACTERISTICS

One of the fundamental goals of drinking water treatment is the removal of particulate matter from influent raw water. By emphasizing particulate removal, multiple objectives are considered (e.g., organic removal, physical removal of pathogenic organisms, sorbed organic/organic removal, and turbidity reduction), which ultimately improves the effectiveness of microbial inactivation by disinfection and reduces the production of potentially carcinogenic disinfection by-products. To provide water of sufficient quantity and quality to consumers, the integrated design of the individual processes must be considered as the effluent of one process becomes the influent of the downstream process. The design process must also account for the desired flow capacity and influent water quality. Additionally, the US Environmental Protection...
TABLE 1  Individual treatment processes included in the three treatment configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Rapid Mix</th>
<th>Flocculation</th>
<th>Sedimentation</th>
<th>Filtration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2  Water quality and flow rate parameters values used as inputs for determining the least-cost treatment regions

<table>
<thead>
<tr>
<th>Influent Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle concentration—mg/l</td>
<td>1.2, 3.6, 6.8, 12, 24, 30, 48, 64, 80, 96, 112, 128</td>
</tr>
<tr>
<td>Shape parameter—c</td>
<td>2.43, 3.55, 4.65, 5</td>
</tr>
<tr>
<td>Sand ratio—mg/m³</td>
<td>0.10-10.0, 0.25-21.0, 0.50-93.0, 0.75-73.0, 1.0-103.0</td>
</tr>
<tr>
<td>Flow rate—mg/hr (MHA)</td>
<td>2.75, 103.75, 75 (203.39)</td>
</tr>
<tr>
<td>Particle density—g/mm³</td>
<td>1.05, 1.20, 2.40</td>
</tr>
<tr>
<td>Temperature—°C</td>
<td>20</td>
</tr>
</tbody>
</table>

Agency (USEPA) requires a cost-benefit analysis when formulating new regulations (Pantolus, 1977), which further emphasizes the need to combine cost estimates, process models, and influent water quality conditions to enable representative national cost estimates to be generated for proposed regulatory actions (e.g., Frey et al., 1998; Garzian, 2001).

Early studies on particulate removal were focused on describing and exploring the effects of an integrated treatment system of rapid mix, flocculation, sedimentation, and filtration. Some of the earliest research to recognize the importance of describing particulate removal with an integrated design was developed by Gross et al. (1973) and Lawler et al. (1980) using different process models. Gross et al. (1973) used a continuous formulation, assumed the influent particle diameters to be of one size, and performed an economic analysis of the sedimentation and filtration costs for treatment. Lawler et al. (1980) assumed ideal plug-flow conditions, described the influent particle diameter distribution with a power law, and investigated a wider range of influent particulate and treatment characteristics. Ramaley et al. (1981) furthered the study of Lawler et al. (1980) by exploiting the treatment behavior for two model waters with different particle densities and performing a sensitivity analysis on the treatment process parameters (mixing intensity, sedimentation residence time, and filter media size). These studies illustrate the complex behavior of particle removal and the need for an integrated design framework to make associated treatment decisions.

Wiesner et al. (1985) and Wiesner et al. (1987) followed the modeling approaches of Lawler et al. (1980) and Ramaley et al. (1981) to explore the least-cost design regions assuming charge neutralization conditions. These studies (Wiesner et al., 1985; Wiesner et al., 1987) took a simplified sedimentation model and optimal flocculation design from least-cost direct filtration solutions as the basis for the conventional sedimentation design; thus, the design process took an endogenous perspective. Wiesner et al. (1987) explored the behavior of systems with contact, direct, and conventional treatment for seven influent particle characteristics and determined the resulting least-cost design regions as a function of particle size, density, and concentration. Wiesner et al. (1987) observed a semi-agreement between actual treatment plant design and the regions predicted by Wiesner et al. (1987). Wiesner et al. (1987) used empirical treatment process models and data from existing treatment plants in a learned formulation that produced an estimate of the influent volume average process cost of removing a specified percentage of suspended solids concentration, and treatment cost parameters. Dharmappa et al. (1994) included a particle size distribution and process models similar to those of Lawler et al. (1980) and Wiesner et al. (1987) for flocculation and sedimentation, and incorporated a hydraulic model based on O'Malley and Ali (1971). The resulting model provided an improved estimate of the least-cost design for three influent conditions. The process models used by many researchers (Dharmappa et al., 1994; Wiesner et al., 1987; Wiesner et al., 1987; Ramaley et al., 1981; Lawler et al., 1980) to describe the changes in particle size distribution assume (1) the particles follow a rectilinear flow path (i.e., particle movement within a fluid is independent of hydraulic effects) and (2) all particles are spherical and nonporous. Studies by Han and Lawler (1991) and Han and Lawler (1992) presented a current modeling approach to account for hydrodynamic effects in the collision processes for spherical, porous particulates. Additional studies have been undertaken to model change in particle size distributions assuming particulate that behave in a fractal nature (Lee et al., 2000; Logan, 1997; 2000; Wiesner, 1992). Therefore, it seems that the hazard risk of sedimentation treatment systems. Chellam and Wiesner (1993),
FIGURE 1 Optimal least-cost values for designs over a range of influent concentrations with an influent particle size range of 0.25–23 μm.

The direct filtration region is the upper region centered on 10 mg/L. The lines (without symbols) represent the added cost into annual costs (CC), operation and maintenance (O&M), and chemical (CHEM) costs (time between the lines). Total cost = CC annualized at 8% over 20 years + annual O&M cost + annual CHEM cost.

\[
\frac{dN}{d\log(d_p)} = 2.3 A d_p^{(1-\beta)}
\]

in which \( N \) is the number concentration of particles, \( d_p \) is the particle diameter, \( A \) is a parameter related to the particle concentration, and \( \beta \) is a parameter related to the shape of the particle size distribution. The particle size distribution is completely specified by selecting a value for \( \beta \) and the particle size range, which allows the determination of the volume average diameter, \( d_{avg} \). The continuous particle size distribution is represented by a logarithmic discretization of particle sizes into 51 discrete particle diameters using \( \Delta \log(d_p) = 0.04 \) (as in Ramaley et al. [1981]). Additional bins are included and initially left empty to account for the formation of particles with diameters up to 300 μm and prevent particle buildup in the largest influent particle size class. The change in particle size distribution through rapid mix, flocculation, and sedimentation is described using Sturrock's equations (Amirtharajah & O'Melia, 1990; Lawler et al., 1980). Removal of particles during sedimentation is incorporated using a term for Stokes' settling (Wiesner et al., 1987; Lawler et al., 1980) including a correction factor for drag on large particle sizes (Montgomery, 1983). The sedimentation tank is discretized into seven layers, which are coupled through the settling time. The particle collision efficiency, \( \alpha \), for the rapid mix, flocculation, and
FIGURE 2 Least-cost design regions and cost contours as a function of the volume average diameter (with the associated \( \beta \) values provided) and influent particulate concentration for an influent particle range of 0.25-25 \( \mu \)m.

Influent Concentration (mg/L) vs. Flow rate (L/min) (27.52 ML/A, \( p = 1.26\) mg/L)

---

The resulting set of ordinary differential equations (ODEs) (63-88) for rapid mix and flocculation and 441-460 for sedimentation, dependent on the influent size range, are a set of ODEs solved using a Fortran solver (Hindmarsh, 1983). The filtration model used was developed by O'Melia and Ali (1976) and uses the volume average diameter, \( d_{av} \) (not the particle size distribution) and particulate concentration leaving the sedimentation tank, \( C_p \), as inputs to the filter model. The filtration model assumes that the highest effluent concentration occurs immediately after the start of a filter run and that operating causes continual filtration improvement. This assumption ignores the deterioration of efficient concentration from particulate breakthrough and, as a result, implicitly assumes that the maximum headloss is reached prior to the occurrence of particulate breakthrough. The filter media properties for this study consist of a media diameter, \( d_{av} \), of 0.1 cm (0.04 in.) and a clean-bed porosity, \( \varepsilon \), of 0.36. The collection efficiency of the sand media, \( \alpha_s \), and entrapped particles, \( \alpha_p \), are 0.76 and 0.08, respectively (Wiesner et al., 1987). The cost equations used to represent the capital costs and operating and maintenance (O&M) costs are taken from Clark (1982) and a modified form of the equations presented by Lerman and Tchobanoglous (1980) to describe filtration O&M costs (Wiesner et al., 1987). Cationic polymer costs (not included in the Clark (1982) O&M costs) are included at a $5.918/cwt ($6.52/kg). The costs are updated to 1987 dollars and amortized over 20 years at an interest rate of 8%. Total cost is presented as the annualized capital cost plus the annual O&M costs, plus the chemical costs. A complete description of the process models is provided in Boccelli (2003).

PROBLEM FORMULATION

Three treatment configurations are considered for evaluating the least-cost design: contact, direct, and conventional filtration. Each configuration uses a combination of the individual treatment processes of rapid mix, flocculation, sedimentation, and filtration. Table 1 shows the individual treatment processes included in each of the treatment configurations. For each set of influent conditions considered, the least-cost design for each of the three treatment configurations is determined. The configuration with the lowest cost is then determined to be the least-cost option.

The cost and process models are incorporated into an optimization formulation to determine the least-cost treatment design. The optimization problem is formulated as follows:

\[
\min \mathbf{C} = \mathbf{C}_C + \mathbf{C}_O + \mathbf{C}_M
\]

subject to individual process constraints

1. The process constraints
2. Bacterial removal (3)
3. Chemical coagulant cost (4)
4. Filter media cost (5)

and integrated process constraints

1. Filter process (6)
2. Residuals removal (7)
3. Filter backwash (8)
4. Filter maintenance (9)

with the objective of minimizing the sum of the capital costs (CC) and operation and maintenance (O&M) costs, subject to altering the appropriate decision variables subject to being
and integrated process constraints, such as polymer costs (C_A, O&M, and others) are not included in the optimization because these costs for any influent conditions (influent particle concentration, B, and others) are the same across the selected geographic region. Instead of optimizing the design without affecting the optimal design, there are four decision variables: cycle tank volume, \( V_p \) (L^3); mixing intensity, \( C_{mix} \); time-treatment, \( \alpha_p \); filter area, \( \alpha_f \). These are constraints set at optimal values such that each tank volume was always selected to maintain a 0.3-s residence time and a 30-m (16.4 ft) height; and the filter depth was 90 cm (36 in) (Boschetti, 2003). The individual process constraints include flocculation residence time, \( t_f = V_p/\bar{Q} \); and mixing intensity, \( C_{mix} = 0.7 \bar{Q} \); treatment residence time, \( t_p = V_p/\bar{Q} \); and filtration loading rate, \( L_f = \left(\frac{B-A}{L_f - \Delta B}\right) \) (Kawamura, 1991; Linnomaki et al., 1987; Montgomery, 1985). The constraints are implicitly functions of all the individual processes and the recycling individual processes must be considered simultaneously in the design process. Integrated constraints are provided for the design of the effluent particle concentration, \( C_{eff} \); and flow rate (Q) = 10 mgd (37.85 ML/d), \( p = 1.20 \text{ g/cm}^3 \). Table 2 shows the extended initial conditions used to determine the least-cost design regions, based on the previous studies of Wissner et al. (1987) and Lawler et al. (1987). The "base case" values in the first part of this study used a flow rate, \( Q \), of 10 mgd (37.85 ML/d) and a particle density (\( p \)) of 1.20 g/cm\(^3\) (similar to Wissner et al., 1987). To explore the effect of flow rate on the least-cost configurations, additional flow rates of 2 and 75 mgd (7.1 and 283.8 ML/d) were investigated with \( p = 1.20 \text{ g/cm}^3 \). To explore the effect of particle density on the least-cost configurations, additional density values of 0.85 and 2.40 g/cm\(^3\) were investigated with \( Q = 10 \text{ mgd} (37.85 \text{ ML/d}) \). Sixteen influent particle concentrations, seven shape parameter values, \( B \), and five particle size ranges were considered in developing the least-cost treat ment configuration regions. Additional constraints and \( B \) values not presented in Table 2 were used where necessary to define the graphical estimation of the least-cost configuration regions.

**RESULTS: BASE CASE**

For the base case conditions of \( Q = 10 \text{ mgd} (37.85 \text{ ML/d}) \) and \( p = 1.20 \text{ g/cm}^3 \), Figure 1 shows a typical least-cost design region.
Influent concentration from 0.5 to 1.5% of the total annual cost.

Last-cost configuration regions. Figure 4 shows the optimal cost contours and least-cost treatment configurations over a range of influent concentrations and \( \frac{L_p}{d_p} \) (with corresponding \( \beta \) values shown) for the basecase conditions \((Q = 10 \, \text{mgd} \, [37.85 \, \text{MIL/d}], p = 52 \, \text{g/cm}^2)\) with an influent size range of 0.25-25 \( \mu \)m. The shape of the cost contours in Figure 2 are representative of the cost contours observed with the other four influent size regions. The cost of treatment tends to read a maximum between \( \beta \) values of 4 and 15. The decrease in cost as \( \beta \) approaches a value of 5 is associated with the increase in particulate concentration in the small particulate sizes. The increase in number concentration results in an increase in the collision frequency, so that treatment is easier and the cost of treatment is reduced. The cost contours for other influent size regions are generally the same, particularly at larger influent sizes \( \frac{L_p}{d_p} \), at which filter run time, not effluent concentration, primarily affects the design decision.

Figure 2 also shows the least-cost configuration regions of contact, direct, and conventional treatment. Contact filtration is typically the least-cost configuration at low concentrations. The influent concentration at which contact filtration is the least-cost configuration increases with an increase in \( \frac{L_p}{d_p} \), when \( \frac{L_p}{d_p} \) is > 2 \( \mu \)m. Direct filtration is typically preferred only over a narrow range of influent concentration, though it is more likely to be optimal at low \( \frac{L_p}{d_p} \) corresponding to high \( \beta \) values. For an influent particle size range of 0.25-25 \( \mu \)m, direct filtration is the least-cost treatment configuration in two small regions corresponding to the small and large \( \frac{L_p}{d_p} \) regions. NonswEEP conventional filtration is the predominant treatment configuration, with the largest ranges at very small particle diameters around \( \frac{L_p}{d_p} \) of 1.5 \( \mu \)m.

When developing the least-cost designs, the common ranges for \( L_p \) and \( d_p \) were allowed to extend beyond typical design values to provide more possibilities in the optimal designs and to allow comparisons between design practice and the optimal results. Typical values for \( L_p \) are 80 L/min/m² (2 gpm/sq ft) and above and for \( d_p \) are 22 \( \mu \)m or less. Figures 3 and 4 show the least-cost design regions and the associated values of \( L_p \) and \( d_p \) respectively, with an influent particle range of 0.25-25 \( \mu \)m. In general, there are two regions where the current design approach produces constraint values below the typical minimum loading rate or above the typical maximum run time: (1) the influent concentration approaches the edge of the cost contour for the conventional treatment.
The least-cost design regions for contact, direct, and conventional treatment and constraint regions (1A, 2A, 3A, 3B) as a function of the volume average diameter (with the associated $\beta$ value provided) and influent particulate concentration for an influent particle range of 0.25–25 μm.

![Diagram of least-cost design regions for contact, direct, and conventional treatment.](image)

**Figure 6.** Cost of filtration as a function of the fiber influent volume average diameter.

![Diagram showing cost of filtration as a function of fiber influent volume average diameter.](image)

Contours represent filtration costs for fiber influent particulate concentrations. $C_r$: signifies the regions of contact and conventional treatment for constraint regions (1A, 2A, 3A). Arrows correspond to increasing plant influent particulate concentration (see inset, some as Figure 1).

Influent size range of 0.25–25 μm is the contact treatment.
The results of a study on the effect of particle size on the least-cost treatment configuration are presented. The least-cost treatment configurations were determined for various influent particle sizes and concentrations. The results are presented in a series of graphs and tables, showing the impact of different particle sizes on the treatment costs and efficiency.

**RESULTS**

The results of the study indicate that the least-cost treatment configuration is significantly influenced by the particle size distribution of the influent. As the particle size increases, the treatment costs also increase. The study recommends using a particle size filter with a high particle retention efficiency to minimize the treatment costs. Additionally, the study suggests using a more efficient treatment method for smaller particle sizes to reduce the overall treatment costs.

**CONCLUSIONS**

The study concludes that the particle size distribution of the influent has a significant impact on the least-cost treatment configuration. The results suggest that using a particle size filter and an efficient treatment method can significantly reduce the treatment costs. The study recommends further research to develop more advanced treatment methods that can effectively treat influent with a wide range of particle sizes.
FIGURE 9. Least-cost regions as a function of influent turbidity and particle size.

FIGURE 10. Cost per unit of effluent for various combinations of influent and effluent particle concentrations.

FIGURE 11. Effect of particle density on the least-cost configuration.

TABLE 1. Design parameters for minimum cost configuration.

As particle density increases, the required filtration capacity decreases with larger plants, and the design parameters shift to lower costs for smaller plants.

RESULTS OF PARTICLE SIZE

The minimum cost for the design configuration is shown in Table 1. The design parameters for the minimum cost are shown in Table 2.

The cost per unit of effluent is noted in Table 3.
Individual and integrated process constraint values from the optimal designs of contact filtration with $C = 4$ mg/L, conventional filtration with $C = 48$ mg/L, and $\beta = 3$ and 4 for various influent particle size ranges, particle densities, and flow rates.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\beta$</th>
<th>$D_{50}$</th>
<th>$f_2$</th>
<th>$D_2$</th>
<th>$L_{1}$</th>
<th>$C_{mean}$</th>
<th>$R_{E}$</th>
<th>$C_{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration</td>
<td>3.0</td>
<td>1.48</td>
<td></td>
<td></td>
<td>74.9 (1.87)</td>
<td>0.63</td>
<td>3.8</td>
<td>62.2</td>
</tr>
<tr>
<td>Concentration</td>
<td>4.0</td>
<td>0.60</td>
<td></td>
<td></td>
<td>129 (2.27)</td>
<td>0.60</td>
<td>8.5</td>
<td>12.0</td>
</tr>
<tr>
<td>Conventional</td>
<td>3.0</td>
<td>1.48</td>
<td>60.0</td>
<td>35.3</td>
<td>126 (3.20)</td>
<td>0.60</td>
<td>2.9</td>
<td>37.9</td>
</tr>
<tr>
<td>Conventional</td>
<td>4.0</td>
<td>0.60</td>
<td>60.0</td>
<td>10.0</td>
<td>126 (3.20)</td>
<td>0.60</td>
<td>4.3</td>
<td>107</td>
</tr>
<tr>
<td>Small size range: 0.75-7.5.0 mm, 1.3 g/cm$^3$, $Q = 10$ mgd (37.85 ML/d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentration</td>
<td>3.0</td>
<td>4.37</td>
<td></td>
<td></td>
<td>413 (10.3)</td>
<td>0.36</td>
<td>2.9</td>
<td>37.9</td>
</tr>
<tr>
<td>Concentration</td>
<td>4.0</td>
<td>1.80</td>
<td></td>
<td></td>
<td>400 (7.73)</td>
<td>0.36</td>
<td>2.6</td>
<td>30.5</td>
</tr>
<tr>
<td>Conventional</td>
<td>3.0</td>
<td>4.37</td>
<td>60.0</td>
<td>34.5</td>
<td>300 (7.73)</td>
<td>0.36</td>
<td>2.6</td>
<td>30.5</td>
</tr>
<tr>
<td>Conventional</td>
<td>4.0</td>
<td>1.80</td>
<td>60.0</td>
<td>34.5</td>
<td>300 (7.73)</td>
<td>0.36</td>
<td>2.6</td>
<td>30.5</td>
</tr>
<tr>
<td>Small size range: 0.75-7.5.0 mm, 1.3 g/cm$^3$, $Q = 10$ mgd (37.85 ML/d)</td>
<td></td>
<td></td>
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<tr>
<td>Concentration</td>
<td>3.0</td>
<td>4.37</td>
<td></td>
<td></td>
<td>414 (10.3)</td>
<td>0.17</td>
<td>2.6</td>
<td>16.0</td>
</tr>
<tr>
<td>Concentration</td>
<td>4.0</td>
<td>1.80</td>
<td></td>
<td></td>
<td>300 (7.73)</td>
<td>0.17</td>
<td>2.6</td>
<td>16.0</td>
</tr>
<tr>
<td>Conventional</td>
<td>3.0</td>
<td>4.37</td>
<td>60.0</td>
<td>28.2</td>
<td>300 (7.73)</td>
<td>0.17</td>
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<tr>
<td>Conventional</td>
<td>4.0</td>
<td>1.80</td>
<td>60.0</td>
<td>28.2</td>
<td>300 (7.73)</td>
<td>0.17</td>
<td>2.6</td>
<td>16.0</td>
</tr>
</tbody>
</table>

Influent concentration, $C_{mean}$ = average concentration, $\beta$ = particle size parameter constraint, $D_{50}$ = particle diameter, $f_2$ = filtration efficiency with $C_{mean}$, $D_2$ = density $D_2$, $L_{1}$ = filtration efficiency with $C_{mean}$. $C_{mean}$ = mean volume concentration, $R_{E}$ = dimensionless response time, $C_{k}$ = concentration at time $t$.

RESULTS: OPTIMAL DESIGNS

Table 3 provides the individual and integrated process constraint values for contact filtration with $C = 4$ mg/L, and conventional filtration with $C = 48$ mg/L over a range of influent conditions. As previously discussed, all of the optimal designs have $(D_{50})_2$ values. The first set of results is representative of the “base case” solutions (particle size range of 0.25-25.0 mm, $\rho = 2.20$ g/cm$^3$, and $Q = 10$ mgd (37.85 ML/d)). For contact filtration, $\beta = 3$ results in a lower loading rate because the associated $(D_{50})_2$ is closer to the particle diameter associated with maximum flotation efficiency (about 2.5 mm). For conventional treatment, $\beta = 3$ is in the 2A constraint region (maximum flotation), whereas for $\beta = 4$ the solution is in the 2A constraint region (minimum flotation).

In both cases, the optimal settling time is large, and these
FIGURE 12 Fraction of particulate matter removed by the filter for different particle densities over a range of influent concentrations for two influent size ranges

\[
\begin{align*}
\text{Influent particle range} & - 0.25-9.5 \mu m \\
\% \text{ Removed by} & \quad \text{Influent concentration (mg/L)} \\
\% \text{ Removed by} & \quad \text{Influent concentration (mg/L)} \\
\hline
\hline
p = 1.25 \text{ g/cm}^3 & 100 & 0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\
p = 1.20 \text{ g/cm}^3 & 98 & 80 & 60 & 40 & 20 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
p = 2.60 \text{ g/cm}^3 & 95 & 85 & 75 & 65 & 55 & 45 & 35 & 25 & 15 & 5 & 0 & 0 \\
\hline
\end{align*}
\]

- In general, increasing the flow rate expands the region in which contact filtration is optimal. Although increasing the flow rate increases the overall design cost, the cost per gallon decreases, illustrating economies of scale for high-capacity plants. Increasing particle density \((p = 1.05, 1.20, \text{ and } 2.40 \text{ g/cm}^3)\) decreases the direct filtration region and expands the contact filtration regions as the least-cost treatment configurations. Although increasing \(p\) reduces the cost of treatment by reducing the size of the sedimentation tank because of more efficient removal of large particles during sedimentation, the actual percentage of particulate matter removed by the sedimentation tank decreases with increasing particle density. The reduction in particulate removal during sedimentation comes as a result of better large-particle removal, which would otherwise continue to coagulate with smaller particles via differential sedimentation, thus decreasing the overall removal of particulate matter from the water.

**ACKNOWLEDGMENT**

This research was performed while Bocelli was a doctoral student in the Department of Civil and Environmental Engineering at Carnegie Mellon University. The authors acknowledge the National Science Foundation.